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3. LEARNING TO TEACH MATHEMATICS

The Lesson De-brief Conversation

In the United Kingdom it is typical for prospective teachers to teach lessons which are observed by a school-based mentor and a university tutor, following which there is a de-brief between those three, about the lesson. In this chapter, we will explore the significance of these de-briefing conversations, within the broader process of becoming a teacher of mathematics. There has been limited attention given to ways of conducting the lesson de-brief within the mathematics education literature, but there are many characterisations of mentoring relationships, which have implications for such discussions. We analyse the practice that has developed at the University of Bristol, making use of fictionalised accounts, based on our experiences. Our particular de-briefing practice appears to be highly effective in allowing prospective teachers to identify and become committed to next steps in their development as teachers. We put forward some tentative reasons for why what we do is effective, linked to our overall enactivist perspective on the process of becoming a teacher of mathematics.

INTRODUCTION

In England, it is typical for prospective teachers to teach lessons that are observed jointly by a school-based mentor and a university tutor, following which there is a conversation between those three about the lesson. We call this three-way conversation the 'de-brief conversation', although similar conversations occur in different countries and are called by different names, for instance, post-lesson discussion; lesson feedback; lesson evaluation. The lessons taught are part of what might be called 'field' experience, or 'practicum' or, in England, we would say a 'school placement'. Our placements in secondary schools form part of a one-year (or 36-week) Post Graduate Certificate of Education (PGCE) in Secondary mathematics (age 11-18 years). The school placements occur at three points during the training year and last approximately eight weeks (in the Autumn), eleven weeks (in the Spring) and five weeks (in the Summer). All four authors teach (or have taught) on the course. In this chapter, we will explore the significance of de-brief conversations that take place between a school-based mentor (a practising mathematics teacher in the school), a prospective mathematics teacher and a university tutor, within the broader process of becoming a teacher of mathematics. These conversations are not only to allow feedback from the university tutor and school-based mentor to the

prospective teacher but are to support new awarenesses about teaching and learning to arise for the prospective teacher. We view the de-brief conversation as a key tool in learning to teach mathematics. Here, we use our reflections on those conversations as a tool for considering how *we* are learning about ways of being a mathematics teacher educator (Chapman, 2008) in a university context, with some responsibility for how de-briefing conversations are run.

In 2014, Even (2014) noted there was little research done on the practice of didacticians, where that label includes those with responsibility for the education of prospective teachers of mathematics. There is an increasing focus, now, on the learning of teacher educators and the skills needed to work with teachers, particularly in the area of facilitating discussion of video recordings of lesson (e.g., Borko, 2014; Gaudin & Chalies, 2015; Karsenty and Arcavi, 2017; van Es, Tunney, Goldsmith, & Seago, 2014). However, there has been relatively little attention given to the orchestration of discussion immediately after a lesson observation (Livesey & Rempe-Gillen, 2014), what we are calling a de-briefing conversation. Before setting out our own approach to de-briefing and our own learning about facilitating these conversations, we review what has been written in the area.

WAYS OF WORKING WITH PROSPECTIVE TEACHERS IN SCHOOLS

A review of work within education on mentoring of prospective and practising teachers coined the phrase "mentoring muddle" (Semeniuk & Worrall, 2000, p.424) to capture the variety of different approaches, definitions and possible purposes for a mentoring relationship. The authors warn of the potential paradox of mentoring programmes, drawing on insights from the work of Britzman:

> Within the obligation to offer programs of change lurks the technocratic impulse, the desire to simplify the complexities of discursive practices and magnify the application of discrete techniques. The danger is that this impulse displaces the slower work of understanding how the immediate fashions our desires. (Britzman 1991, p. 239)

The importance of respecting the length of time, within this "slower work of understanding" is a theme that recurred through our study of the literature on work related to de-briefing conversations. Our aim in this chapter is to respect the complexity of the conversations we are studying and point to ways of working productively with this complexity, rather than searching for discrete techniques.

In one of the few early studies in this area, Halai (1998) reflected on her own approach to mentoring of practising teachers and concluded that the challenge of successful mentoring is simultaneously to:

- address the issue of teachers' lack of mathematical knowledge and understanding,
- establish a relationship with the teachers which was open and built on trust, and
- help the teachers gain insights into alternative approaches to teaching (p. 311).

Issues around mentoring have subsequently been considered within reviews of teacher professional development and learning, both broadly and within mathematics education. Timperley, Wilson, Barrar, and Fung (2007) considered evidence for what occasions sustained learning for teachers. Of relevance to the focus of this chapter was the conclusion, in relation to contexts of professional learning, that the extremes were generally ineffective, of either assuming teachers are self-regulating professionals who just need time and space to reflect on their teaching, or of having 'experts' and other professionals offering prescriptive practices (even with a rationale) for teachers to follow. Rather:

> Effective communities provided teachers with opportunities to process new understandings and challenge problematic beliefs, with a focus on analysing the impact of teaching on student learning (p. xxvii).

The de-briefing conversations between an educator, school-based mentor and prospective teacher recur at points throughout any teacher education course in England and therefore can be viewed as one element of a community; the importance of processing understanding, challenging beliefs and focusing on student learning are all, therefore, significant. From our perspective as mathematics teacher educators we would also want to add in a category that captured something about focusing on mathematical content. Each of us sees part of our role as mathematics teacher educators as supporting prospective teachers to come into a more complex relationship with the subject mathematics, than when they arrive on our course. For many of our prospective teachers, this will involve an "un-packing" of previously learnt algorithms that have perhaps never been questioned, expanding their "highly compressed understandings" (Ball & Bass, 2000) in preparation for working with learners' expressions of mathematics. There is an important process also of opening up to new and alternative methods and ways of seeing particular mathematical concepts, perhaps with the question of "why?", which has not been asked before, in turn leading to opening up to alternative teaching strategies.

A review of 106 articles within mathematics education about the professional learning of teachers identified 39 studies that focused on teachers' collaboration and community (Goldsmith, Doerr, & Lewis, 2014). Intriguingly, the authors commented that they were often not able to distinguish between when studies were focused on peer collaboration from studies focused on more of a mentoring relationship with an expert (which they classify as "coaching"). Most of these studies were small scale and the difficulty in determining the kind of collaboration involved points to the lack of attention that has been given to the specifics of conversations following a lesson observation. Goldsmith et al. (2014) summarise their findings in this area by concurring with Timperley et al.'s (2007) comment that "participation in some form of learning community was a necessary but not sufficient condition" for teachers' learning (p. 88).

Differences between the terms (used above) of coaching and mentoring are complex and contested. Halai (2006) suggests that mentoring tends to be defined by the roles played by mentors and that key roles include: "guide and, provider of support and key information" (p. 702). Halai (2006) then views coaching as "a form

of in class support to provide (novice) teachers with feedback on their functioning and thereby stimulate reflection and self-analysis" (p. 702). As noted by Goldsmith et al. (2014), coaches might be peers or experts. We do not seek to make a clear distinction between mentoring or coaching and focus more on the kinds of relationship and conversations that might and do occur between prospective teachers, school-based mentors and university tutors. We note that Halai (2006) pointed, at that time, to a gap in research around roles mentors could or do play.

Jaworski (2003) has conducted work at the interface of practice and theory, within mathematics education. Jaworski offers the concept of "co-learning" that, in her work, is used to define one possible mode of interaction between a researcher and a school practitioner, collaborating and reflecting on actions via "mutually sustaining roles and goals" (p. 250). The concept of co-learning is used to describe practices in China under the umbrella term of "lesson study" (Huang, Su, & Xu, 2014). Post-lesson discussions are part of the lesson-study process and mentoring discussions have been subject to a detailed and systematic analysis (Gu & Gu, 2016). In their summary of past work on the content of effective mentoring discussions, post lesson (and including work within Japanese Lesson Study), Gu and Gu identify six elements:

> presenting new knowledge from research and curriculum, discussing mathematics and pedagogy, reflecting on teaching and learning, providing teachers with constructive feedback, attending to student learning, and facilitating productive discussions (p. 444).

These elements match closely a study in the United States, looking at the content of conversations between mathematics teachers and coaches (Mudzimiri, et al., 2014). Little is known yet about how to develop the skills to make such coaching discussions effective. Takahashi (2013) points to the significance of the "knowledgeable other" in the context of Japanese lesson study, in terms of post-lesson conversation, again concluding that more work is needed to understand how to develop the expertise needed to support lesson study effectively.

One of the few conceptual contributions (within mathematics education) *about* research on mentoring is a recent study, in the context of a professional development programme, that considered the detail of discussions between a teacher and teacher educator (Heyd-Metzuyanim, Munter, & Greeno, 2018). This study pointed to the importance of the concepts of "framing" and "meta-rules" in discussions between teachers and those supporting them (and reports on a case where frames were not aligned). A meta-rule is linked to the idea of a socio-mathematical norm (Yackel & Cobb, 1996) and captures the significance of patterns in the activity of a group (that may or may not be made explicit). These patterns might be specific to particular ways of doing mathematics (e.g., when dividing by a fraction) or more generally about the discourse within a group (e.g., the pattern of initiation-response-evaluation found in some classrooms would be a meta-rule that is generally not made explicit). Heyd-Metzuyanim et al. (2018) then define a frame as lying somewhere between specific and general meta-rules and to be: "based on regularities that can be observed in repetitive human interactions" (p. 26). The concept of a frame draws on the work

of Goffman (1974) who recognised the way that in every human interaction there are vital communications *about* the interaction that help participants interpret and respond appropriately. These meta-communications (usually not verbalised) are the frame.

Relating now to our precise concern in this chapter, the situation of a prospective teacher, school-based mentor and university tutor is not specifically considered in any of the studies reported above. It may be that this situation is relatively unique to an English context, something which in itself perhaps warrants further investigation. However, we feel the issues raised in this chapter are relevant to any kind of mentoring relationship, whether in a pair or three (and indeed in two of the stories we discuss, the school-based mentor was not in fact present). Where this particular form of triad has been investigated (Wood & Turner, 2015) the context was an analysis of student work, rather than a joint lesson observation. An important conceptual idea in this study however, relevant to de-briefing conversations, is that of a hybrid or third space (Moje et al., 2004) in which prospective teachers, practising teachers and university tutors generate contexts that resist hierarchies between the kinds of knowledge that might typically be associated with a 'practising' professional and an academic. We are exploring the hybrid space of the de-briefing conversation. We will be drawing on the notions of frames and meta-rules and discussing the content of conversations.

META-RULES FOR DE-BRIEFING CONVERSATIONS AT THE UNIVERSITY OF BRISTOL

Whilst each de-briefing conversation is different, we find that comments from observers (such as, for example, those acting as external examiners for the course) frequently identify a sameness in the conversations. This points to certain common characteristics of these conversations and we will discuss examples of these. Similar observations of common characteristics are made about prospective teachers completing the course, with school-based mentors talking about qualities of self-awareness and awareness of their practices, consistent with the explorations in, and of, such hybrid spaces. We share a practice of taking detailed verbatim notes of what we hear and see in lessons, avoiding judgments. We have a consistent focus on the mathematics in a lesson and will often work on the tasks in a lesson, together with the school-based mentor, during the lesson, looking for different ways of approaching the topic, for example. We recognise shared norms between us, as authors, in our awareness of the importance of the prospective teachers developing listening skills so that they can respond to their students.

Although the support needed by prospective teachers is to develop a way of being in the classroom that they value in supporting the learning of their students, as part of the process, issues can be created by prospective teachers not listening that arise in multiple and complex ways. We want to hear from prospective teachers about what they notice. However, if there are principles, such as listening and hearing, that are not being carried out when the university tutor and school-based mentor observe the lesson, rather than simply pass that comment on, the practice developed of

asking, “What have I got to offer as an action that can be carried out by a particular, prospective teacher in future lessons so that prospective teachers use themselves in different ways?”. For instance, a teacher who seems to be taking up all the space in a classroom might be challenged to teach a lesson in silence, as in the second of the two cases that follow here.

INTERVENTIONS AS A MATHEMATICS TEACHER EDUCATOR (LAURINDA)

Case 1: The need for mathematical intervention

As a university mathematics teacher educator, I was involved in a cascade project in a large country. This involved a central team taking responsibility for direction and organisation. Each area of the country had a local manager who had a team of advisory teachers who in turn visited schools and ran professional development sessions. I was the consultant. Myself and the lead of the whole project observed a lesson given by a teacher who applied the rule "two negatives make a plus" to examples of the form -2 -5 to get +7. The project aims had been, in de-brief conversations, for the advisory teacher to start with inviting the teacher to tell an incident from the lesson. The head teacher of the school had asked to observe our interactions. As soon as the advisory teacher had spoken, the head commented: “You need to teach my teacher some mathematics, that is the most important thing.” So, we did. In fact, as can be seen in the next text, if the head teacher had not intervened, we would have felt it to be important that we worked on some mathematics.

Case 2: The need for interventions with teaching strategies

What seems to be important is that the individual recognises what they consider to be positive changes in the behaviour of their pupils in response to different behaviours in themselves. What follows is one paradigmatic story of how I was able to give a prospective teacher an action to take.

> *Before*: Visiting a prospective teacher, she seemed to fill all the spaces in the classroom both physical and mental. After the lesson, she talked about how the children were not learning anything and how shocked she was at what was written in their books. It was definitely their “fault” they didn’t listen. I wanted her to do a lesson where she was silent throughout. She looked shocked, “You mean after I’ve started them off?” “No, throughout.” And left.
>
> *After*: On my next visit she played the function game (Brown and Waddingham, 1982, p. 42) with the children. The game was used by the departmental team. The game is played with the teacher in silence. I especially remember her eyes being so prominent as she threw them about the room. She was smiling and attending, seeming larger than life, more theatrical than previously. To decide what to do rather than what to say involved her in responding and attending to what the pupils were doing and, although there were no words, she learnt to listen in that session.

In the rest of this chapter, the current three Post Graduate Certificate of Education tutors on the course at Bristol create other such stories, in this case fictionalised accounts (Hannula, 2003) to bring out practices and principles underpinning lesson observations and the following de-briefing discussions.

AN ENACTIVIST FRAMING

Our approach as mathematics teacher educators arises from an enactivist framing. Building on the ideas of Maturana and Varela (1987), enactivism places an emphasis on knowing rather than on knowledge and on the equivalence of knowing and doing: "all doing is knowing, all knowing is doing" (p. 27). In this framing, cognition is not determined or constrained by an internal structure; rather, "the structure of the individual coemerges with this world in the course of, and as a requirement for, the continuing inter-action of the individual and the situation." (Reid, Dowden, Jeans, & d'Entremont, 2000, I-10). This structure, whilst continually changing and being changed by interactions with the people and interactions that constitute the individual's world, determines responses to triggers through a history of structural coupling (Maturana, 2002). In working with prospective teachers and their school-based mentors, we seek to make de-brief conversations spaces in which to support and guide a process of "deliberate analysis" (Brown & Coles, 2012), allowing "reconstruct[ion of] the intelligent awareness that justifies the action" (Varela, 1999, p. 32). Compared with expert teachers, these prospective teachers will have access to a significantly restricted set of awarenesses but, by challenging automatic responses, such a process of deliberate analysis can support the development of a rich and subtle set of possible actions (Brown & Coles, 2011).

Within an enactivist framing, it is perhaps inevitable that such analysis begins with an invitation to re-enter the experience of the lesson. We refer to this as giving an "account-of", following the pattern established by Mason (1987) and Jaworski (1990) but conversation that remains at the level of behaviours is ineffective in expanding possible actions, since any alternative behaviour will remain disconnected from the next classroom situation. Part of the role of the mathematics teacher educators in this context, then, is to enable the focusing of attention on "purposes" (Brown, 2005), questions that are likely to arise from specific experiences, but which have relevance over time and which permit evaluation of further behaviours as effective or not. We refer to this process of shifting attention from an experience to an issue (a "purpose") and possible future actions as giving an "account-for", relating interpretations to what was seen, trying out possible meanings and explanations (Watson & Mason, 2007, p. 40).

The same process informs our conversation and collaboration as mathematics teacher educators and, indeed, shapes the structure of this writing. Here, too, we look to reconstruct "the awarenesses that led to action" (Brown & Coles, 2011, p. 862) in order to expand the set of possible actions. In what follows, we offer four stories from our practice, interspersed with an interlude of reflections on becoming a mathematics teacher educator. These are fictionalised accounts (Hannula, 2003), to

preserve anonymity but drawing on combinations of real events. The stories illustrate points that have stayed with us, whether through recognition of sameness or of difference when compared with what we might often experience. They are not to be read as models for de-brief conversations but are offered in order to provoke reflection. You might choose to attend to what each story triggers in you as you read it, before turning to our accounting for each.

ASKING A QUESTION AND COFFEE AND CAKE (LAURINDA)

The class seemed full of tall adolescents and there were so many of them. She got lost in amongst them even though she was their teacher. Her teacher presence, so clearly there with her year 7s earlier in the year, had deserted her. "When's sir coming back from paternity leave?"; "We've got our exams in a few weeks after Easter?"; "Why have they given us you?".

Talking about the lesson and listening to her observations simply seemed a waste of time. She was upset. I found myself asking a question, "So, what's different about teaching this group than being with your year 7s?" It was a question for which I did not want a quick surface answer. "Do you know Naffeteria? Down the Gloucester Road? Meet you there for the de-brief and I'll buy you coffee and cake."

Accounting for

Sometimes that's all that's needed. Someone to listen to and hear the venting of frustration and upset. Those year 11s had been taught by the same teacher for the last three years and they want him back. The next year 11s taught will not feel so big because they have been known for longer and will likely have been taught by the teacher for longer, and she won't be a prospective teacher, she will be developing a teacher persona. She will literally be looked up to even if the students are taller. For now, a ritual de-briefing is not what is wanted, nor talk about teaching strategies. Sometimes the context conspires against you even after many years' teaching. What was important was that the prospective teacher calmed down and was able to see some of these issues and was still prepared to go back into the classroom the next day, to learn about teaching from the younger classes and to survive with this examination- year class of adolescents. I am not sure that any of Gu and Gu's (2016) categories were appropriate.

DIARY ACCOUNT OF A DE-BRIEF (TRACY)

This is a story from my first year as an mathematics teacher educator.

> The prospective teacher (Hayley, a pseudonym) taught a group of year 9s. Top set. The topic was simultaneous equations. There were lots of good elements of the lesson and I remember thinking what a nice class they were. This was my first time talking to Hayley, we met on a day at the university mid-placement, but she was quiet and didn't speak during the group session that I

remember. We had an hour together straight after the lesson without the school-based mentor who then joined us after that hour.

I guess I didn't have any expectations about how the feedback might go – but I think I was surprised that she spoke negatively about the behaviour of the group of students – perhaps because I had been so aware of my emotional reaction to the class – as such a positive one? So, we didn't start with what went well! Which I became quite quickly aware of and was keen to focus on. She didn't seem to think anything went well – she even said she had thought about it and couldn't come up with anything. I said what I thought was something positive. Perhaps because the mentor was not there. I think I felt a bit uncomfortable about doing this – I suppose because I have talked and thought about my role during a de-brief conversation. This was different in that the mentor was not there and in that she was struggling to see a positive. My strategy was to turn to my notes from the lesson and read them out, hoping that she might see a positive in what she had been saying.

How much is it my role to state the positive? I am conscious of not casting a judgement – is judgement bad? How much of this is because I have been told judgement is bad? Is there a place for making positive judgements/statements? As a teacher I would offer praise as part of my practice so where does the praise come in this role? I remember saying how sophisticated her ability was in reflecting on what she might do differently next time and her awareness of what she would like to improve.

I was not sure if she was looking for praise or even believed it. I think she valued it. Was I uncomfortable? I don't really think I was massively uncomfortable, but I was aware of conflicts in what I had talked about doing and observed Alf doing and what I was doing – subtle conflicts.

We talked about jobs, Hayley had decided to apply for jobs in the independent sector. We talked at some length about her reasons for this. To begin with this seemed to be about behaviour. I tried to unpick this with her, again without being positive or negative about state vs. independent but I was intrigued about the reasons behind this decision. I was also aware that I was being reminded of a teacher who I worked with years ago who became very negative about the students that he worked with and ultimately left teaching before completing his induction year. I wanted to challenge this negativity towards students. Through talking, Hayley was able to unpick this negativity and talk about what she did that she would like to have done differently and that the issue of students' poor behaviour was actually one of a lack of motivation.

Accounting for

Hayley appeared to be stuck with her image of a poorly behaved class. My awareness of *not* knowing how to act in that moment, triggered by feeling uncomfortable, led

me to read the verbatim notes I had taken from the classroom observation, the notes were to hand and so I began to read. This bringing the conversation back to a place of what was said in the lesson (an account of) seemed to release Hayley from being stuck with her image. Through talking about particular moments in the lesson, she began identifying possible alternatives within her own behaviour and re-labelling the issue or purpose as one of a lack of motivation, this suddenly felt like something positive we could work on, and I could now see the issue for myself (whereas I was unable to see any issue of poor behaviour). There seems to be a mix in here of most of Gu and Gu's (2016) categories, and the discussion certainly felt productive.

A CONDENSED DIALOGUE OF A DE-BRIEF CONVERSATION (JULIAN)

This is a fictionalised account of the conversation between the prospective teacher (Colette), associate tutor (Jack) and Julian following a joint-observation of a 60-minute lesson with a Year 7 class (ages 11 and 12) of 30 pupils on the topic of "Rearranging equations". (Teachers' names are pseudonyms). The fictionalised observation was conducted in the second week of the second school placement of the year.

Julian: So, you know that we will start with you. Let's hear about planning to action… Talk about your planning and how that moved into what happened in the lesson.

Colette: OK. Well, we always start in the same way, with numeracy questions. That is in silence and it is on a timer. Everyone is used to that. I knew I needed to ask them about new exercise books, so I put that in the plan, but then I couldn't find them, so that took some time. But it was OK, because students were working. I'm still getting used to collecting scores from the numeracy starter. We [Colette and Jack] had talked before about needing to get everyone's attention, so I had planned what to say.

Julian: What did you say?

Colette: I think I said, "Can you put your pens down and look this way."

Julian: That's what I have written down, too. So…

Colette: So I collected in the scores and introduced the starter task. I wanted to have a reminder of what happens when multiplying positive and negative numbers. I wasn't sure how they would deal with multiplication of the letters.

Julian: What did you notice about what they did?

Colette: Well, they could tell me the right answers.

Jack: Yes, the people answering your questions could do it. What about the others?

Colette: Hmm… well… yes, I'm not sure.

Julian: OK, so we might want to come back to that as an issue at the end. Move us on through the lesson.

Colette: I knew that the class had done some work last week [with the class' usual teacher] on balancing equations. So I wanted to start from an equation that was obviously true and get them to manipulate it.

Julian: Can you talk us through the example you used?

Colette: Well, I had my slide prepared with "2+3 = 5" and I asked someone to choose a number between one and ten, then wrote down "add 4 to both sides" and asked someone to evaluate it. They told me it would be worth nine, so I wrote that, too. We did it again, but subtracting and we got a negative number, which was nice. They seemed pretty confident with the idea, so I changed the slide and got them to copy down the key point about doing the same thing to both sides of an equation to keep it balanced. I had to ask them to be quiet then.

Jack: Where were you at this point?

Colette: I was moving around the room, seeing what people were doing.

Jack: Right. So you might want to think about your routines for establishing what noise level you expect.

Accounting for

This account was created from notes of more than one de-brief and it draws together themes that arose across those conversations. One such theme is the location of the energy in the comments made. As the prospective teacher re-enters their experience of the lesson by giving their account of events, I was aware of listening for indications of when their talking indicates being on the cusp of a shift, a challenge to the teacher's frame of mathematical learning. Once I have noticed an emergent shift, my expectation would be stay with it, to explore the discomfort and encourage the teacher to draw out an issue. Here, I have related an experience when I did not do this, instead marking the point in the conversation ("We might want to come back to that as an issue at the end") and then asking the teacher to return to the detail of their experiences of the lesson. My decision came in response to a directive question from the school-based mentor ("What about the others?"), offering something to the prospective teacher from an awareness that they had in that moment. The roles taken by the three participants in the de-brief conversation, (school-based mentor, university tutor and prospective teacher) move with some fluidity between the elements identified by Gu and Gu (2016). In this account, there is an indication of differences in emphasis of the elements of reflecting on teaching and learning, providing constructive feedback and facilitating productive discussion. In fact, my awareness of the school-based mentor offering a modified meta-frame led me to an action I would not have anticipated: moving away from the point of traction and continuing with the account of the lesson. The partial mis-alignment of meta-rules (Heyd-Metzuyanim et al., 2018) between the school-based mentor and me in the

conversation highlights another purpose in these de-brief conversations: as a university tutor I am working with the prospective teacher but also (perhaps chiefly) working with the school-based mentor, looking to maintain a consistency at the meta-level of the course (Brown, Helliwell & Coles, 2018), a theme developed further in the next section.

INTERLUDE: WORKING TOGETHER AS MATHEMATICS TEACHER EDUCATORS

As part of the rhythm of the mathematics teacher education course at the University of Bristol, we meet as a group of mathematics teacher educators and discuss events that are planned for the day and again after these events, to reflect together on experiences, identify issues and plan actions. We also meet to discuss issues that have stayed with us over longer periods of time, often with reference to writing about these issues. In one such meeting, when sharing our fictionalised accounts of de-brief conversations, the most experienced member of the group, Laurinda, commented on a strong sense of difference between the account presented by the most recent addition to the team, Julian, and the practices that were established in the initial design of this mathematics teacher educator programme. This comment sparked a response (from Julian) that revealed a desire to maintain sameness. Such a desire seemed at odds with what had been perceived from the writing, but it resonated with the current course leader, Tracy, who described a sense of coming out of a similar phase after working on the course for two years. A few days later, Julian met with Tracy to continue the discussion. This extended conversation was recorded and transcribed and what follows are three sections from the conversation which relate to de-briefing, where Tracy and Julian capture something of the changes they are experiencing, as they become more experienced in the mathematics teacher educator role. We offer each section of transcript and then reflect on what we learn about becoming a mathematics teacher educator, one of the themes of this chapter.

De-brief conversations as a beginning Mathematics Teacher Educator

[Transcript conventions: … indicates some dialogue missed out; (.) indicates a pause; standard punctuation is used where possible to help convey meaning.]

Tracy: I went to a school to see Alf visit one of his students before I did any of my visits. It was those three things which I kind of knew having been a school-based mentor and having had conversations about where we begin. I don't remember it being done to me, those three things. It's more recent than that. But I think I was doing it myself as a school-based mentor.

Julian: Just for clarity, could you just articulate again what the three things are?

Tracy: I'll say something like, 'Let's begin where we normally begin. So that's what went well, what not so well, what would you do differently'

Julian: There's a sense in which my approach to these de-brief conversations was trying to produce a certain sameness. And it transpires that I haven't,

which I think is what triggered quite an emotional response for me in that moment … this idea of trying to have sameness, produce sameness of experience, I think I'd probably want to say continuity of experience, for PGCE students on the course from year to year and flowing on. That's the sense that I had of this stage of wanting to maintain sameness.

Tracy: When I first began here, just over two years ago, I wanted rules, like a recipe. And the 'what went well, what not so well, what would you do differently' was a recipe which it still is to a certain extent. I suppose I am only now getting to the point where I feel confident to work with what's actually happening in the moment, rather than what I think I *should* be doing. Laurinda will comment sometimes in passing and I'll think about these comments and they'll have quite a profound effect on me. One that's really stuck with me, is, "your relationship is with the mentor", which I might not have got to on my own. My interpretation of her comment might be very different to what she meant, but I think my learning as a teacher educator comes through the process of me making sense of the comment through doing de-brief conversations rather than trying to come up with some sort of right answer for what it means I *must* do. So, it's not that there's a right or wrong way. I think two years ago I would have wanted to know, "Well, what does that mean? How do I do that?" whereas, through doing de-brief conversations I am forming my own conviction about "your relationship is with the mentor". After all, it is the mentor that I'm going to have an ongoing relationship with, over many years in some cases, and that's the person who is going to have the biggest impact on the prospective teacher. If I can work with the mentor, that means that they'll be able to work with their prospective teachers.

Julian: There seems a sense in which you've done that thinking and it releases your awareness to be on working with what's happening.

Tracy: Yeah, and also a sense that I have to make this work. I have to be, happy with what I'm doing because otherwise I don't want to do it anymore, it is really hard to try and conform to one way of being. That certainly doesn't mean it's not helpful to think what would Laurinda or Alf do in any situation, because sometimes I need a strategy, but I also need to be convinced in my own right that what I'm doing is the right thing, I suppose. So I just am different, I have a different way of doing things but I am open to take on board the pointing out of differences and then work on them. I suppose the difference now from two years ago is that the pointing out of differences would have triggered a strong desire to be the same and maybe now the pointing out of differences just offers me, a different way to be. So I don't necessarily change what I'm doing but I have opened up the possibility of acting differently. If I see what I'm doing is not working then I'll probably go back to a conversation I've had … Does that make sense?

Julian: It's sort of increasing the space of the possible.

Tracy: Yes, it is.

In the same way that Laurinda talked about ways of supporting prospective teachers in developing their awareness of the children in their classrooms, by paying attention to what the children are saying, Tracy is talking about developing her awareness in order to work with what is happening in the moment of the de-brief conversation with prospective teachers. As a beginning mathematics teacher educator, Tracy's awareness of not knowing how to act led to her need for a "recipe" or a structure to follow. In using the structure and then reflecting on the de-brief conversations, Tracy was able to work towards releasing her awarenesses to work with what the prospective teachers are saying in the moment. Only through doing de-brief conversations has Tracy begun to find her own "conviction" about how to act based on what is happening, based on what she has come to know from her past experiences of these de-brief conversations and through making sense of offerings from more experienced mathematics teacher educators. The following section of transcript further develops the idea of moving towards conviction in the context of becoming a mathematics teacher educator:

> Tracy: I'm well aware that from personal experience I do have to feel a strong sense of what I'm doing for my own reasons. So that line we say so often, "there's no one model of good mathematics teaching", doesn't mean that anything goes but it's about supporting students to find their own way, their own conviction in what they're doing, becoming the mathematics teacher they can become, otherwise why would they stay in the profession? It certainly doesn't mean never offering them anything, because I do that regularly, but there has to be a need for it, something from within the awareness of the prospective teacher. That's why when I sometimes say, "Oh, why don't you try this or that", there is no apparent uptake, because it hasn't come from something within the prospective teacher's awareness. So, it brings me back to the fact that what I offer on its own is of less importance than working with the person's awarenesses. Find something *they* want to do differently and then work on it, because they are developing their own convictions about what they want their classroom to look like. So of course, I offer them different ways of being, but I can't force that. In the beginning I wanted their classrooms to look how I used to want them to look in my own school, I had to work hard on myself to not to let this longing get in the way of seeing what was there in any lesson I was observing and work with the prospective teacher's awarenesses.
>
> Julian: I have a sense, then, that now there's a feeling of 'I want this teacher to look like how I want a teacher to look.' Not in the sense of having one model of teaching but in the sense that you're saying, of having a sense of their own conviction which feels to me like you're expressing something about the philosophy of the course.
>
> Tracy: Which goes back to the "not one model of good mathematics teaching", because if there isn't *one* model of good teaching, and there are many, as many

> as there are people doing it. I see my role as guiding them in finding their own way given what is happening in front of their eyes.

Having had a strong conviction as a teacher of mathematics and a Head of Mathematics in the way that mathematics could be taught, Tracy talks here about being aware early on of a desire for her prospective teachers to create classrooms that fitted her own image of mathematics teaching from being herself a teacher of mathematics and the need to hold back from this desire. This desire is reminiscent of what Pimm (1993) calls "teacher-educator-lusts":

> I think we should examine equally critically our *need* (lust?) for the teachers we work with to change. Their change is not our business; how, when and if they change is surely their concern alone… If I as a teacher educator can only feel successful if the teachers I work with change (and in ways I want them to), I am setting up both myself and the teachers I am working with quite dramatically. (p. 31)

Of course, this in not to imply mathematics teacher educators should become laissez-faire and if a prospective teacher is unaware of key things happening in their classroom to the point, for example, that children are not safe or not learning, then some quite direct intervention and interpretation will be necessary. But, in becoming a mathematics teacher educator, Tracy is finding strength of conviction through supporting prospective teachers to find conviction in their own teaching through developing images of their own classrooms. Conviction is not found through the words of others alone, Tracy has come to find conviction as an mathematics teacher educator through exploring what is meant by principles such as "there is no one model of good mathematics teaching" in the context of her own experiences of visiting different schools and different prospective teachers of mathematics. Only through these experiences is she able to find meaning.

In the final section of transcript Tracy and Julian discuss a moment from a de-brief conversation where the focus was on working with the prospective teacher on the conceptual development of some mathematics from a lesson that Tracy had observed. This section illustrates a different dimension of the de-brief conversation where Tracy sees her role as supporting the prospective teacher in 'unpacking' mathematical concepts as a way of reflecting on the lesson taught and supporting the planning of subsequent lessons.

> Tracy: I often try and work with the mentor during the lesson observation on some mathematics from the lesson. This might be an interesting question that a student has asked, or something that has come out of what the prospective teacher is offering or maybe where some mathematics could go further. In one particular lesson, I spent more time than I would usually, working with the mentor on the representations being offered by the prospective teacher. I think what triggered the conversation was a shared sense of unease.
>
> Julian: Unease?

Tracy: Yes, I think we agreed that we weren't really sure what the children would be seeing in the mix of representations that were being presented by the prospective teacher.

Julian: Can you say more about what you mean?

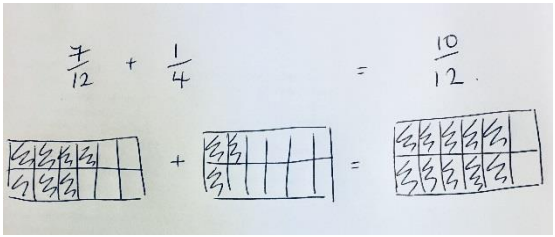
Tracy: Well, it was adding fractions, and there was a moment where the representation offered to the class moved from partially shading fractions of two distinct rectangular wholes coming together as one partially shaded rectangular whole having added them together. There was a moment where we looked at one another and I just asked the mentor what they were seeing in the image that was on the board at the time. We just started working on it together.

Julian: And that was during the lesson?

Tracy: Yes, something I have done more and more of I think is take these opportunities so that in the three-way de-brief after the lesson it is likely to come up and be productive. It is also something the mentor might do more regularly, that is, think about the mathematics during the lesson so it can be discussed later.

Julian: And did it come up in this case?

Tracy: Yes, it did. I think the mentor just had this on a piece of paper



and he asked the prospective teacher, what do you see in this image? We then worked together on what each of us saw before moving to considering any limitations and alternative representations. Somehow, the prompt coming from the mentor was more powerful than it coming directly from me. I think that is something that I have come to know.

There are often situations that arise where something significant is not yet within the prospective teacher's awareness and the conversation recalled above illustrates Tracy looking for opportunities to develop or even force these awarenesses through working both with the mentor, and with the prospective teacher.

The final story of a de-brief comes from Alf who, at the time of writing, had been working in the mathematics teacher educator role for eight years and so was in some intermediate position, in relation to experience and conviction, between Laurinda and Julian/Tracy.

A CONDENSED DIALOGUE OF A DE-BRIEF CONVERSATION (ALF)

Prospective teacher (Kyle), associate tutor (Bryony) and Alf sit down as a three straight after the lesson. (Pseudonyms)

Alf: So, it is the questions I am sure you are always asked, what went well, what didn't go well, what would you have done differently if you had your time again? But just take it slow and start somewhere.

Kyle: Well I was happy with the start, they all seemed quite calm and especially for a last lesson in the day and I was actually quite pleased with what they had remembered about how to solve equations.

Alf: Okay, let's pause on what was happening that meant the start went well – because I think Bryony and I would agree with you.

Bryony: Yes, it was very calm.

Alf: So, what were you doing that made it calm?

Kyle: I've always got something on the board when they come in, but I've just started giving them all a sheet of paper as they come in, when it's this lesson of the week [last lesson of the day], with a task printed that they can write on. And I try now to make it something we did last lesson that we will be using in this lesson, so hopefully they all know what to do and they can just get straight on with it as soon as they come in.

Alf: Yes, that feels right. Bryony and I were also struck by the way you gave students feedback on that starter task, getting answers from the students and just writing up what they told you.

Kyle: That's something I've doing for a while now and I like to try and go through one question all together, just to reinforce what they should all be able to do.

Alf: That's lovely, so something there about the power of routines and what strikes me is you having routines within routines. So, you have this overall routine that you will have something on the board and you have a lesson "patter" and, within that, you have a structure for what kinds of things you will do for different lessons in the week and you have a way you always give them feedback. And all this just adds to the sense that students came into the room knowing what is expected of them. So, are you doing this with all your classes?

Kyle: No, that's definitely something I could improve with my other classes. This one is probably the one that is most routine-ised. They were my hardest class at first but now I actually think they are going better than the others.

Alf: Right, so you have evidence there for the power of routines.

Bryony: You have really relaxed with this class and it feels like they have accepted you – I don't feel like I need to be in the room.

Alf: So what routines might you put in place for your other classes?

Kyle: Well I guess, the starter on a sheet of paper is definitely something I could try with my year 10 class.

Bryony: I think they'd respond well to that, once a week I still give my year 10s a 'basic skills' starter task. And have you seen the 'five-a-day' resources we sometimes use?

Alf: Great, any other strategies or routines you want to implement, either with this class or other ones? Keeping going with the routines in this class definitely feels important.

Kyle: Yes, and I could do more of the building on last lesson with my other classes as well.

Alf: Right and part of that is so that students themselves get a sense of their own learning and progress – "I can do this, and I know I couldn't have done it 2 weeks ago". And that's something you can reflect back to them as well.

Accounting for

It is apparent from my second comment that Bryony and I have discussed the lesson and possible areas for focus. Conversation would routinely take place at the back of the classroom between me and the school-based mentor, in order to check and agree on any elements of feedback that the mentor particularly wants to offer and, more generally, their evaluation of different parts of the lesson. It is a common experience that I need to slow down prospective teachers in their reflections on the lesson and in my second comment (above) I recognise wanting to focus initially on the first thing they say, in this case that they were happy with the start of the lesson. From my perspective as a mathematics teacher educator, I have no idea why they might be happy but I have a conviction that if they are able to articulate the reasons, this may lead to some useful awarenesses of things they could be doing more often or with other classes. For the prospective teacher to make a comment about the start there must be something noteworthy, that is, perhaps this is something they have worked hard on recently or that they have had difficulty with in previous lessons. My comment to pause the conversation and focus on the first thing said is a meta-comment and serves as a framing of the conversation, focusing perhaps on something close to Gu and Gu's (2016) reflecting on teaching and learning.

In this conversation, Kyle articulates some practices he has implemented successfully with this class, that I label as 'routines' and reflect back to Kyle that the fact of these patterned behaviours is likely to be a reason he feels his relationship with this class has improved. The label 'routines' is not a new category but perhaps here can be linked by Kyle in a new way to actions he is aware of using with one class but not others. The possibility opens up, in this conversation, to extend practices that are effective with one group, to other groups that he teaches. I am also conscious of attending to student learning (Gu & Gu, 2016) and for opportunities to

push prospective teachers to consider how they could support their students to be aware of their own learning. In other words, I recognise an awareness I have of a typical transition in the journey towards becoming a teacher, that starts with initially assessing student learning and monitoring student understanding with a focus (for the prospective teacher) on the prospective teacher's own actions. Over time, this shifts into paying attention to how students might become aware of their own learning and take responsibility for monitoring their own progress. This conversation provides an opportunity to raise this possibility with Kyle but without the expectation that this will necessarily be heard by him.

CONCLUSION

In this chapter, we have explored different ways of engaging with prospective teachers in conversations following a lesson observation, we labelled these 'de-brief' conversations. In England a de-brief would typically occur between the prospective teacher who has taught the lesson, a university tutor (mathematics teacher educator) and a school-based mentor, although the school-based mentor may not always be present. While there is a vast literature on mentoring in education, there has been little work specifically within mathematics education and even less that is focused on what takes place following a lesson observation. Given that lesson observations, and the targets and judgments arising from them, are key elements of learning to teach (in England, and other countries) this seems a surprising gap.

What we have aimed to articulate is a particular approach to de-brief conversations that has been developed at the University of Bristol and we have offered stories that aim to capture examples of the kinds of conversation we have following a lesson. We hope to have demonstrated in these stories that these conversations create spaces in which to provoke and process new awarenesses relating to the teaching and learning of mathematics and, indeed, of the mathematics itself, with a continual focus on the learning of the students with whom the prospective teachers are working. In doing this, we have tried to pay attention also to offering a sense of how we, too, are in a process of learning, just as much as the prospective teachers (and, indeed, the school-based mentors). Our learning is about the learning of the prospective teachers. Their learning is (we hope!) about learning to teach mathematics. The delicate shift, as a mathematics teacher educator, between using the practices of another mathematics teacher educator and having conviction in our own way of working, is explored in the interlude via the interview between Julian and Tracy; as is the tension between personal conviction and working towards some common convictions on a course for prospective mathematics teachers.

The stories we have offered above do not illustrate all that is occurring in de-brief conversations. We might, for example, have presented further accounts of work on mathematical concepts. This might arise as a focus from difficulties students in the classroom had with a particular idea, or perhaps simply from our awareness as mathematics teacher educators of alternative approaches to a concept to the one offered in the classroom. As described above, we might well work on some mathematics with the school-based mentor, at the back of the room during the lesson

we are observing, and part of our aim there would be to establish a meta-rule for communications that the concepts of mathematics are never far from focus. A focus on mathematical concepts is an element of mentoring relationships not described in current literature.

A theme for each of us is the development of awareness and, in particular, awareness that allows us to act. In Laurinda's story that led to conversation over coffee and cake, an awareness of the implications of the prospective teacher's affective state, making it near to impossible for the teacher to access possible issues, led to a re-positioning of the de-brief conversation, physically and emotionally. In her story of visiting Hayley, Tracy highlights an awareness of not knowing what to do. This awareness of a 'not' also leads to an action, returning to the detail of the lesson, which Tracy identifies in the interview with Julian as indicative of a particular stage on a journey to release her awareness in subsequent de-brief conversations. Julian's story illustrates an awareness of difference of frame with the school-based mentor, leading to an action that postponed working on a possible issue so that there was space to privilege the prospective teacher's own experience and account. This decision was driven by another awareness – that 'telling' is not enough. In Alf's story of working with Kyle we see Alf's awareness of the prospective teacher needing to slow down in order to identify experiences that lead to action, with the prospective teacher not being aware of their own account at a meta-level. Alf also relates his awareness of a staging post in the prospective teacher's development, where talk shifts from focusing on what the teacher does (the 'I' in the account of assessment) to what students are doing.

Whilst there is some commonality in most of the fictionalised accounts of de-brief conversations presented here, the fact that there are not discrete techniques in evidence across the stories may be taken as reflecting the approach of the course overall, which came into focus during the conversation between Tracy and Julian: that there is not one good model of mathematics teaching. This chapter certainly does not offer a template for de-briefs or mentoring relationships, but we have seen how similarities in approach have emerged through conversation, reflection and converging conviction. As noted previously, not all de-brief conversations in the context of this course follow the same structure. Rather, through returning to certain ideas, shared between university tutors, a sense of conviction has emerged for each tutor that is harmonised by reflecting together. As Tracy put it,

> I know when I come away from a de-brief and think, 'That felt alright,' or if I come away and have a bit of an uncomfortable feeling and need to go and talk about it [with the other university tutors] or think about it a bit more. But I think I've just experienced enough de-briefs so that I can think more about applying meaning.

Starting from a 'recipe', in this case a three-question structure, gives a way of working. Early work as a university tutor might well remain with the structure, ensuring that there is a pattern that leads to the identification of issues and then to actions that support working on those issues. It is through living this experience repeatedly that conviction emerges in the central ideas, so that starting with what

went well ceases to be something to do and becomes internalised as significant and an expression of values, to be held as a possibility for the beginning of the conversation, or not, as appropriate. Through experience and reflection, these ideas move from something merely done to something done in order to allow change to occur and it is in this movement that we develop awarenesses that allow us to hear and to act. Discussing and writing about experiences has led, in our experience, to an alignment of these awarenesses and the emergence of common characteristics of our work as university tutors. As a group of university tutors, we have experienced resonances in the stories offered in this chapter that lead us to identify a number of such characteristics. Without seeking to be exhaustive, these include the recording of verbatim notes in the lesson, used subsequently to support the de-brief conversation; working with prospective teachers' awarenesses, (i.e. working with existing awarenesses and developing of new ones) in order to support the identification and articulation of convictions; staying with the detail of experiences until issues can be identified in ways that allow for the formation of actions; developing targets for action from a focussed discussion around particular issues that have arisen from within the prospective teachers' awareness; working with the school-based mentor (in and out of lessons, over time, in different ways); working as a group of three; working on mathematics.

Whilst recognising the emergence of common characteristics, there is something powerful in not-documenting the initial structure of the de-brief conversation (we do not have, for example, a list of questions always asked). In not doing so, whilst also reflecting together on experiences of de-brief conversations, space is created to develop convictions out of which to act. This is tempered by the oral history of the course, through the offering of pieces of wisdom, to be taken up in different ways at different stages. An example is the tenet that de-brief conversations are essentially about working on the relationship with the mentor. This seems, on the face of it, at odds with the de-brief process, in which the prospective teacher is invited to reflect on their experiences. What this means, then, has emerged for each university tutor through experiences and shared reflection. Tracy speaks about moving from trying to work out 'how to do that' to maintaining a focus on the relationship with the mentor as an awareness during school visits.

It is perhaps not surprising that our development from an initial focus on what to do, moving towards awareness that allows action through a process of reflecting on experiences in order to identify issues and actions, embodies the approaches that are fostered in the mathematics teacher educator course for our prospective teachers. As a group of university tutors, we engage in learning from experience in the ways articulated above and we structure the course sessions so that those who are becoming mathematics teachers do these same things. For us as mathematics teacher educators and for our prospective mathematics teachers, there is a clear statement that there is no 'best' model, a meta-rule for communication which promotes the development of personal conviction in each of us. Overall, this chapter has aimed to open up debate about the running of de-brief conversations with prospective teachers of mathematics.

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